Factor Analysis: An Overview in the Field of Measurement

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ABSTRACT

Purpose: This article provides an overview of factor analysis from the perspective of measurement in clinical research.

Summary of Key Points: Factor analysis is a statistical technique that identifies interrelationships among a set of items in an instrument and/or questionnaire and groups them into homogeneous domains. Exploratory factor analysis can be used to reduce the number of items in a questionnaire and identify its underlying domains. Confirmatory factor analysis can be used to test a hypothesis about the domain structure of a questionnaire. Principal component analysis and common factor analysis are the most common techniques and differ based on the amount of variability that is analyzed among items. Steps of the factor analytical process include assessing correlation matrices, factor extraction, choosing the number of factors to retain, assessing the factor loading matrix, factor rotation and factor interpretation. Because no standardized method exists, factor analysis involves decision-making at each step.

Conclusions: The different techniques and methods of factor analysis each have unique strengths and limitations. Clinicians and researchers reviewing articles on factor analysis should ensure that authors state a priori their purpose, conceptual approach, preferred technique and methods that will guide their decision-making.

Key words: confirmatory factor analysis, exploratory factor analysis, factor analysis, principal component analysis

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RÉSUMÉ

Objectif: Cet article fournit un aperçu de l'analyse factorielle de la perspective de la mesure des résultats dans la recherche clinique.

Résumé des points clés: L'analyse factorielle est une technique statistique qui identifie des interrelations parmi une série d'items dans un instrument et/ou un questionnaire et groupe ceux-ci en domaines homogènes. On peut utiliser l'analyse factorielle exploratoire pour réduire le nombre d'items dans un questionnaire et identifier ses domaines sous-jacents. Une analyse factorielle confirmatoire peut être utilisée pour vérifier une hypothèse sur la structure du domaine d'un questionnaire. L'analyse en composantes principales et l'analyse des facteurs communs sont les techniques les plus courantes, celles-ci différant en fonction du degré de variabilité des items analysés. Les étapes du processus d'analyse factorielle sont les suivantes : évaluation des matrices de corrélation, extraction des facteurs, choix du nombre de facteurs à conserver, évaluation de la matrice de la charge factorielle, rotation des facteurs et interprétation des facteurs. Étant donné qu'il n'existe aucune méthode standardisée, l'analyse factorielle nécessite la prise de décisions à chaque étape.

Conclusions: Les techniques et les méthodes différentes d'analyse factorielle ont chacune leurs points forts et leurs points faibles. Les cliniciens et les chercheurs examinant des articles sur l'analyse factorielle doivent s'assurer que les auteurs indiquent leur objectif a priori, leur approche conceptuelle, leur technique préférée et les méthodes qui orientent leur décision.

Mots clés: analyse des composantes principales, analyse factorielle, analyse factorielle confirmatoire, analyse factorielle exploratoire

Imagine you are a physical therapist who would like to measure the health of your patients at different stages of rehabilitation. After careful review of the existing literature, you were unable to locate a specific health instrument and decided to develop a new questionnaire. After consulting with patients and health professionals in the field, a list of 20 items was generated for inclusion in the questionnaire. How do you know whether any of the items are redundant and should be removed? How would you go about removing items while ensuring that

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important items remain part of the questionnaire? How do you know what domains are measured within the questionnaire and what items comprise those domains? Finally, how would you go about naming these domains?

Factor analysis is a statistical technique used in the field of measurement that can address the above issues. The goal of this article is to provide an overview of factor analysis in the field of measurement that may be relevant to clinicians and researchers who are interested in learning about the concept of factor analysis to interpret and review articles that use factor analytical techniques. Specific objectives are to discuss (1) the concept of factor analysis and its conceptual approaches and techniques and (2) the steps of the factor analytical process and considerations when reviewing an article on factor analysis. The factor analytical process is demonstrated using a sample data set of 250 responses to a hypothetical, 20-item questionnaire.

WHAT IS FACTOR ANALYSIS?

In factor analysis, individual questionnaire items are grouped into homogeneous domains that represent common characteristics.[1] Factor analysis can be used to reduce the number of items in a questionnaire by identifying the underlying structure of items in a data set and determining whether the data can be explained by a smaller number of domains. Alternatively, it can be used to test hypotheses about a data set's underlying structure. A factor is a linear combination of items that represents an underlying domain.[2] For example, consider a questionnaire with items asking about bathing, dressing and toileting. Factor analysis may yield methods to linearly combine responses to these items into a factor that represents one domain (self-care) of a larger construct (health). Factor analysis commonly includes two major analytical approaches (exploratory and confirmatory) and two major techniques: principal component analysis (PCA) and common factor (or principal factor) analysis (CFA). The term *factor* is used to refer to both a CFA factor and the analogous PCA component. Other important terminology is defined in the glossary.

Conceptual Approaches to the Factor Analytical Process

The two main conceptual approaches to factor analysis are termed *exploratory* and *confirmatory*. When the underlying structure of a data set is unknown, exploratory factor analysis can determine which domains comprise a construct of interest.[3] However, researchers usually have some idea of a construct's composition from previous theory or clinical experience. Confirmatory factor analysis tests hypotheses that state the number of factors that represent the data and the items that comprise each factor.[4] Although you might have ideas stemming from clinical experience about what domains comprise the construct of health from the above example, if uncertain, you may pursue an exploratory factor analysis approach. An exploratory approach is illustrated using the sample data set to uncover the domain structure (if any) of the hypothetical questionnaire. This approach will demonstrate the decision-making required throughout the entire factor analytical process.

Factor Analytical Techniques

To identify the underlying domains of a construct, factor analysis analyzes the variance (or variability) among items in a data set. Common variance is that shared among a set of items that can be explained by a common set of factors.[5] Specific variance is that unique to an item and not explained by common factors.[5] Error variance is that attributed to random measurement error.[5] Generally, there is no way to separate specific variance from error variance; hence, the two are commonly referred to as unique variance. The way in which components of variance are analyzed distinguishes between the two main factor analytical techniques, PCA and CFA. Principal components are linear combinations of item responses that represent common and unique (specific and error) variance. In contrast, common factors represent only common variance (Figure 1).

Although both PCA and CFA allocate items into a smaller number of domains,[6] assume linear relationships among items in a questionnaire[7] and extract components or factors that are uncorrelated,[3] the methods differ conceptually. PCA assumes that components are a linear deterministic combination of item scores. Thus, the analysis moves "forward" from item scores to yield components based on their shared variance (common and unique). In contrast, CFA assumes that factors are latent (unknown hypothetical) variables that causally influence the item scores. Item scores are derived from a linear combination of factors (that represent common variance) plus the item's unique variance. Thus, the analysis moves "backward" from item scores to determine the underlying factors (see Figure 1). Both techniques are illustrated with the sample data set.

Communality and eigenvalue estimates help determine whether a factor exists and which items "relate" to that factor. Communality (h^2), a feature of items in a data set,

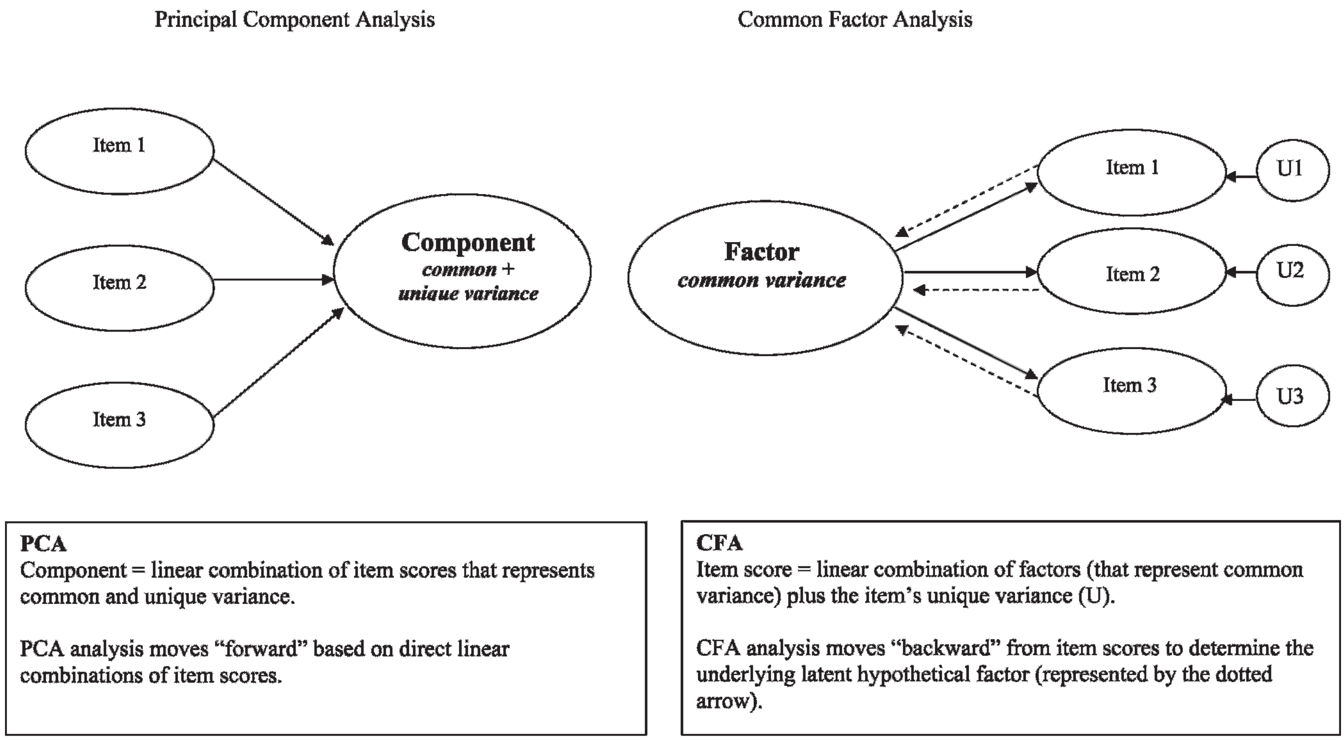

Figure 1. Conceptual difference of principal component analysis (PCA) and common factor analysis (CFA).

represents the amount of variance each individual item has in common with all other items.[4] Communalities range between 0 and 1. High final communalities (close to 1), calculated from the results of a factor analysis, suggest that factors explain the variance better than the individual items. Initial communalities are the preliminary estimates used during the analysis. Initial PCA communalities represent the total variance of each item (common and unique) and are assumed to equal 1.0. In contrast, CFA communalities represent only a proportion of the total variance (common) and are estimated as being less than 1.0.

Eigenvalues (λ), a feature of factors, are defined as the amount of variance in all items explained by a given factor or component.[8] Dividing an eigenvalue by the total variance yields the proportion of variance explained by a given factor. Factors with larger eigenvalues account for greater variance compared with factors with lower eigenvalues. Positive eigenvalues indicate that a correlation matrix is "positive-definite" and thus considered factorable, whereas negative eigenvalues indicate that a correlation matrix is "ill-conditioned" (not factorable) and eigenvalues close to 0 suggest that proceeding with factor analysis is questionable.[3] Given that there are no clear guidelines that suggest when factor analysis should be halted, this value should be determined a priori. PCA eigenvalues are estimates of the total amount of variance, whereas with CFA, eigenvalues are estimates of the common variance among items accounted for by a given factor. As a result, the sum of eigenvalues (and communalities) with PCA is equal to the number of items in the questionnaire, and the sum of eigenvalues and communalities with CFA is less than the number of items in the questionnaire.[3]

FACTOR ANALYTICAL PROCESS

An understanding of the steps and decision-making that occur throughout the factor analytical process can assist when interpreting or reviewing a research article that has used factor analysis. An overview of factor analysis steps is illustrated in Figure 2 and discussed below.

Assessing Correlation Matrices

Authors reporting on factor analytical techniques should assess whether proceeding with factor analysis is feasible by examining the correlation of items in a questionnaire. A correlation matrix consists of correlation coefficients that represent responses for each of the items in a questionnaire. Values in any given row consist of correlations between that item's responses and all other item responses in the questionnaire. Correlation matrices are symmetrical,

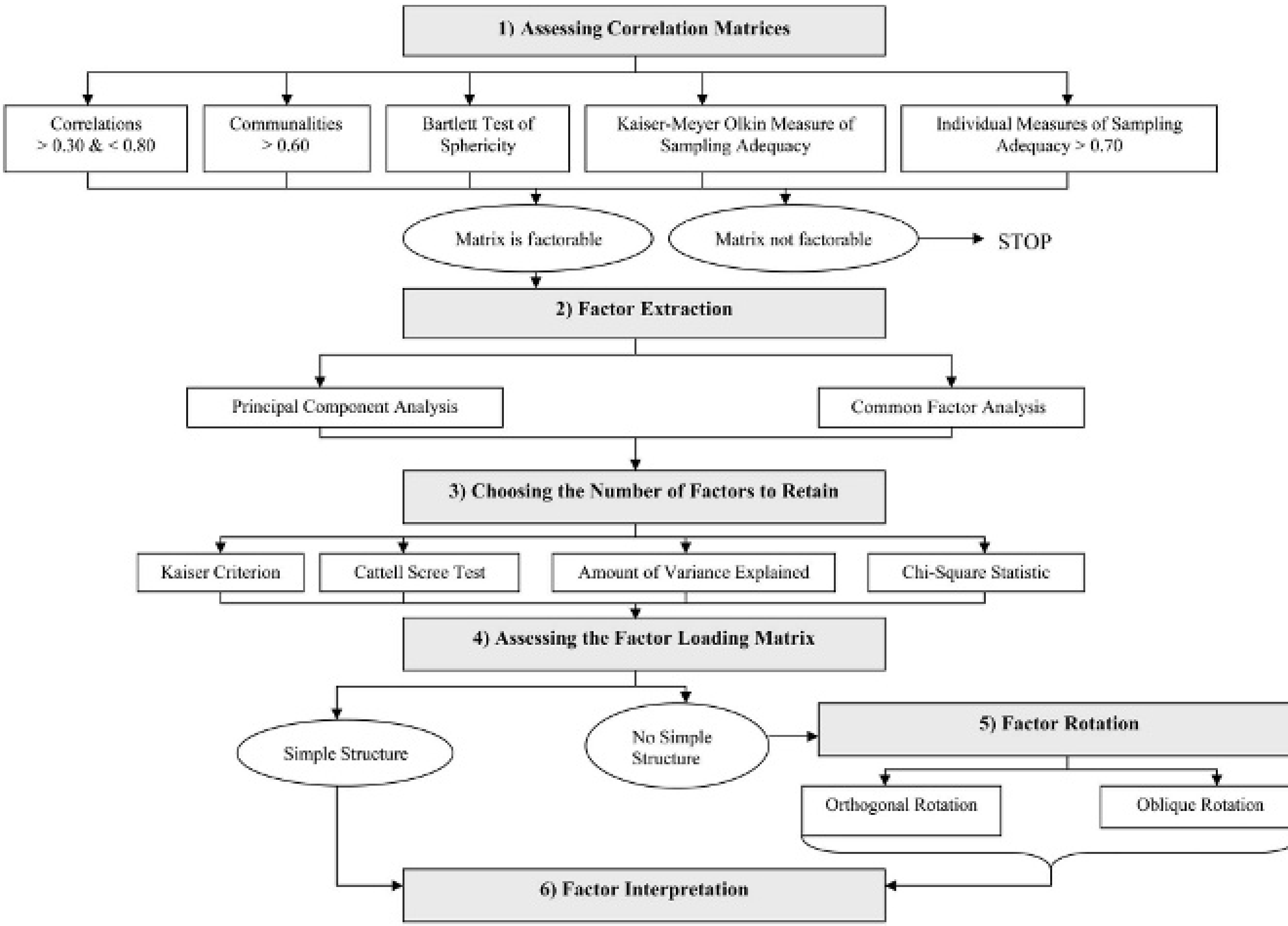

Figure 2. Steps and options available in the factor analytical process.

with values below the diagonal mirroring those above (Table 1). The diagonal of the matrix consists of initial communality values, equal to 1.0 with PCA but must be estimated with CFA because they are unknown. A common estimate (usually less than 1.0) is the squared multiple correlation of each item with all other items.[3]

Five methods to assess correlation matrices are discussed (see Figure 2). The first method directly assesses the correlations and their clinical relevance. Highly correlated values above or below the diagonal (greater than 0.80) suggest that these items may be redundant and could be excluded,[3] whereas weakly correlated values (less than 0.30) indicate a lack of shared variance, a requirement for factor analysis. Authors should also assess correlations for clinical relevance. For example, a question about walking should correlate more highly with a question about climbing stairs than with one about memory.

The second method assesses the initial communalities within a correlation matrix. Values greater than 0.60 are desirable, indicating that the shared variance is enough to suggest that the matrix is factorable.[8]

The third method, the Bartlett Test of Sphericity,[9] uses statistical methods to test the hypotheses that the correlation matrix is an identity matrix (a matrix with no relationship among the items, with all diagonal values equal to 1.0 and all off-diagonal values equal to 0). If authors are unable to reject this hypothesis, the test indicates that factor analysis may not be safe. However, because large sample sizes tend to result in the hypothesis commonly being rejected, this test should be used in combination with other methods.[10]

The fourth method, the Kaiser-Meyer Olkin Measure of Sampling Adequacy (KMO),[11] also assesses whether a correlation matrix is an identity matrix but does so by comparing original squared partial correlations with those calculated after the effects of all other items are removed. KMO values above 0.90 suggest that an item is ‘‘marvelous’’ at contributing to the interrelationship with other

Table 1. Correlation Matrix (n = 20 questionnaire items)

Item	*1 Pain*	*2 Energy*	*3 Arm Strength*	*4 Leg Strength*	*5 Walking*	*6 Stairs*	*7 Eating*	*8 Dressing*	*9 Laundry*	*10 Cleaning*	*11–16*	*17 Feel Afraid*	*18 Forgetfulness*	*19 Concentration*	*20 Overall Health*
1 Pain	**1.0**	0.59	0.59	0.61	0.69	0.49	0.49	0.59	0.55	0.54		0.54	0.47	0.46	0.57
2 Energy	0.59	**1.0**	0.70	0.75	0.67	0.55	0.50	0.55	0.57	0.59		0.49	0.52	0.52	0.55
3 Arm strength	0.59	0.70	**1.0**	0.76	0.54	0.53	0.56	0.51	0.53	0.45		0.38	0.38	0.39	0.52
4 Leg strength	0.61	0.75	0.76	**1.0**	0.75	0.70	0.39	0.45	0.45	0.50		0.35	0.50	0.49	0.60
5 Walking	0.69	0.67	0.54	0.75	**1.0**	0.72	0.40	0.52	0.56	0.56		0.39	0.40	0.45	0.58
6 Stairs	0.49	0.55	0.53	0.70	0.72	**1.0**	0.35	0.40	0.52	0.52		0.43	0.48	0.49	0.49
7 Eating	0.49	0.50	0.56	0.39	0.40	0.35	**1.0**	0.49	0.48	0.50		0.35	0.37	0.41	0.51
8 Dressing	0.59	0.55	0.51	0.45	0.52	0.40	0.49	**1.0**	0.62	0.62		0.54	0.57	0.40	0.58
9 Laundry	0.55	0.57	0.53	0.45	0.56	0.52	0.48	0.62	**1.0**	0.63		0.49	0.38	0.39	0.58
10 Cleaning	0.54	0.59	0.45	0.50	0.56	0.52	0.50	0.62	0.63	**1.0**		0.50	0.39	0.52	0.55
11 Groceries	0.60	0.57	0.46	0.56	0.78	0.52	0.53	0.57	0.58	0.61		0.35	0.45	0.48	0.61
12 Work	0.42	0.43	0.34	0.41	0.66	0.60	0.42	0.60	0.55	0.62		0.53	0.51	0.60	0.52
13 Take transport	0.54	0.61	0.45	0.57	0.66	0.51	0.52	0.59	0.61	0.64		0.42	0.55	0.56	0.66
14 Feel lonely	0.52	0.57	0.41	0.42	0.41	0.50	0.42	0.51	0.55	0.45		0.60	0.65	0.56	0.57
15 Feel sad	0.56	0.57	0.36	0.43	0.37	0.52	0.32	0.52	0.51	0.49		0.72	0.62	0.68	0.75
16 Feel anxious	0.34	0.39	0.32	0.39	0.34	0.39	0.38	0.39	0.47	0.46		0.75	0.65	0.50	0.44
17 Feel afraid	0.54	0.49	0.38	0.35	0.39	0.43	0.35	0.54	0.49	0.50		**1.0**	0.64	0.48	0.59
18 Forgetfulness	0.47	0.52	0.39	0.50	0.40	0.48	0.37	0.57	0.38	0.39		0.64	**1.0**	0.75	0.52
19 Concentration	0.46	0.52	0.40	0.49	0.45	0.49	0.41	0.40	0.39	0.52		0.48	0.75	**1.0**	0.65
20 Overall health	0.57	0.55	0.52	0.60	0.58	0.49	0.51	0.58	0.58	0.55		0.59	0.52	0.65	**1.0**

All correlations significant at $p < .0001$.
Derived from a sample data set of 250 responses to a 20-item hypothetical questionnaire.
Shaded and bolded area represents the diagonal of the correlation matrix. Values above the diagonal mirror the values below.

items, between 0.80 and 0.90 is "meritorious," between 0.70 and 0.80 is "middling" and less than 0.60 is "mediocre," "miserable" or "unacceptable."[11]

A final method for assessing correlation matrices, the Individual Measures of Sampling Adequacy (MSA),[3] assesses each item using only its simple and partial correlation coefficient. Ideally, MSA values should be greater than 0.70, indicating that correlations among the individual items are strong enough to suggest that the correlation matrix is factorable.[3] Items with poor sampling adequacy should be removed from the analysis. With no clear criteria to prefer one method versus another, authors should declare a combination of methods prior to analysis to determine whether a matrix is factorable. Using multiple methods will enable authors to side with the majority when conflicting results may emerge.

For the sample data set, a combination of methods was used to assess the correlation matrix. Correlations among all items are greater than 0.31 and less than 0.79 (see Table 1). Each item correlates with at least one other item (> 0.30), and there are no highly correlated items (> 0.80) in the matrix. Initial communality estimates on the diagonal are assumed to equal 1.0 for PCA and are greater than 0.60 for CFA (not shown). The overall measure of sampling adequacy (KMO) is 0.95, and individual MSA values are greater than 0.70 for all items (not shown). Together, these tests suggest that the matrix is factorable.

Factor Extraction

In factor extraction, item responses are grouped into factors (or components) based on linear combinations that represent the amount of shared variance among the items (see Figure 2).[12] PCA and CFA may yield a similar number of factors (or components), particularly when CFA communalities are high (close to 1.0) and there are multiple items in the data set.[5,6] Both techniques may also use the principal axis method[13,14] to extract factors, a successive process wherein the first extracted factor accounts for the highest proportion of variance in a data set, the second extracted factor accounts for the second highest proportion of remaining variance and so forth, until the last factor explains the smallest amount of variance, making it the least important for explaining the underlying structure of a data set. Other factor extraction methods, such as the maximum likelihood method,[15,16] unweighted least squares, generalized least squares and alpha factoring, are not discussed because of their rare use, complexity, predisposition to yield communalities greater than 1.0[17] and tendency to result in too many retained factors.[3]

Although PCA is perceived to be more straightforward and easily understood,[3] some believe that the CFA approach of extracting factors based only on common variance will result in a more accurate solution.[18] Unless the variance in the first few factors is considerable, PCA may overestimate correlations among items and subsequently overinflate the size of components.[14] In such situations, authors may conclude that components explain more of the underlying structure of a data set than reality.[1,19] However, CFA also has limitations. The squared multiple correlation coefficients, often used to estimate initial communalities, represent the extent that items share variance with each other rather than with underlying factors. Hence, initial communality estimates might not reflect the true common variance or communality of a data set.[3] Another drawback of CFA is that this technique may yield final communalities greater than 1.0.[5] Such results imply that more than the total variance is explained by a given factor, making the correlation matrix difficult to interpret. Lastly, because CFA factors are only estimates of hypothetical variables assumed to influence item scores (opposed to deterministic values based on direct linear combinations of item scores with PCA), individual patient factor scores cannot be calculated.[3] None of these issues were evident in the sample data set.

Although authors would normally determine a factor extraction technique a priori, both are included in the example to illustrate the potential differences between the PCA and CFA results (Table 2). Factor extraction results of the sample data set with PCA yielded 20 components, one for each item (see Table 2). CFA extraction yielded 10 factors (because any factors above 10 possess eigenvalues equal to or less than zero, meaning that the explained variance is nil or negative). CFA yielded fewer factors than PCA because they account for only the common variance. The sum of CFA eigenvalues is 15.49, less than the number of questionnaire items because they represent only the common variance. The proportion of total variance accounted for by a given PCA component is equal to the eigenvalue of that component divided by the total variance of the data set (20.00). The proportion of common variance accounted for by a given CFA factor is equal to the eigenvalue of that factor divided by the sum of common variance (15.49, which excludes unique variance) (see Table 2). Note also that the majority of variance is explained by the first few factors (or components), whereas the remaining factors contribute minimally to the description of the underlying data structure. Rarely will authors retain all extracted factors. How the number of factors is chosen for retention is described in the next step.

Choosing the Number of Factors to Retain

Several methods exist for deciding the number of factors to retain to represent the underlying domain structure of a data set (see Figure 1).[4] The Kaiser criterion[20] retains factors with eigenvalues greater than 1.0, which account for more of the total variance than a single item. Although this is a commonly used method, eigenvalues near 1.0 may pose difficulty. A limitation of the Kaiser criterion is that the results will depend on the number of questionnaire items. A large number of items (more than 50) can result in a greater number of factors that meet the criterion, whereas a small number of items (approximately 20) can result in too few factors.[21] Accuracy may be heightened when a questionnaire consists of fewer than 40 items and the sample size is large (e.g., greater than 10 participants per item).[22]

The Cattell Scree Test[23] plots eigenvalues against each factor (Figure 3). The scree of the plot is that part of the line where the slope changes from negative to almost zero, indicated by a sharp break in the curve. The test, which retains factors prior to the scree, is most accurate when communalities are high and the ratio of items to factors is at least 3:1.[22] A drawback of the scree test is the need for subjective interpretation, which may be complicated when multiple changes in the slope exist or when no obvious change in the slope is apparent.

Assessing the amount of variance explained retains factors until a predetermined threshold of variance is attained. Although no universal guidelines for a threshold of variance exist, one suggestion is to extract those factors that account for 90 per cent of the explained variance, or until the last factor accounts for a small proportion of the explained variance (less than 5 per cent).[24] Authors should state an a priori threshold, which should be lower with CFA compared with PCA since only the common variance is analyzed.

Statistical methods can test whether the number of factors chosen to explain the data set, compared with a model in which items load on one or more factors, is adequate. Authors should begin by testing whether one factor is sufficient and then gradually increase the number of factors until a non-significant result is achieved.[25] An

Table 2. Factor Extraction Results for Principal Component Analysis and Common Factor Analysis

	PCA			CFA				
Factor	*Eigenvalue*	*Proportion of Total Variance Explained*	*Amount of Total Variance Explained (Cumulative)*	*Factor*	*Eigenvalue*	*Proportion of Common Variance Explained*	*Amount of Common Variance Explained (Cumulative)*	*Amount of Total Variance Explained (Cumulative)*
1	**11.885***	**0.5942**	**0.5942**	1	**11.590**§	**0.7483**	**0.7483**	**0.5795**
2	**1.263***	**0.0632**	**0.6574**	2	**1.203**§	**0.0777**‖	**0.8260**	**0.6396**#
3	**1.132***	**0.0566**†	**0.7140**‡	3	0.893	0.0576	0.8836	0.6843
4	0.820	0.0410	0.7550	4	0.585	0.0378	0.9214	0.7136
5	0.668	0.0334	0.7884	5	0.415	0.0268	0.9482	0.7343
6	0.604	0.0302	0.8186	6	0.406	0.0262	0.9744	0.7546
7	0.539	0.0270	0.8456	7	0.236	0.0152	0.9896	0.7664
8	0.500	0.0250	0.8706	8	0.081	0.0052	0.9948	0.7704
9	0.404	0.0202	0.8908	9	0.051	0.0033	0.9981	0.7730
10	0.366	0.0183	0.9091	10	0.028	0.0018	1.0000	0.7744
11	0.304	0.0152	0.9243	Total	**15.488**††			
12	0.278	0.0139	0.9382					
13	0.245	0.0122	0.9504					
14	0.201	0.0100	0.9604					
15	0.193	0.0096	0.9700					
16	0.171	0.0086	0.9786					
17	0.145	0.0072	0.9858					
18	0.130	0.0065	0.9923					
19	0.094	0.0047	0.9970					
20	0.058	0.0029	1.0000					
Total	**20.000****							

CFA = common factor analysis; PCA = principal component analysis.
Bold and shaded rows represent factors retained using the Kaiser and amount of variance criterion.
Derived from a sample data set of 250 responses to a 20-item hypothetical questionnaire.
* Kaiser criterion for PCA: three factors retained with eigenvalues > 1
† Proportion of total variance explained by Factor 3 in PCA: 1.132/20 = 5.7%
‡ Amount of total variance explained by the first three factors in PCA: 71.4%
§ Kaiser criterion for CFA: two factors retained with eigenvalues > 1
‖ Proportion of common variance explained by Factor 2 in CFA: 1.203/15.488 = 7.8%
Amount of total variance explained by the first two factors in CFA: 64%
** Sum of total variance in PCA: equals the number of items in the questionnaire
†† Sum of common variance in CFA

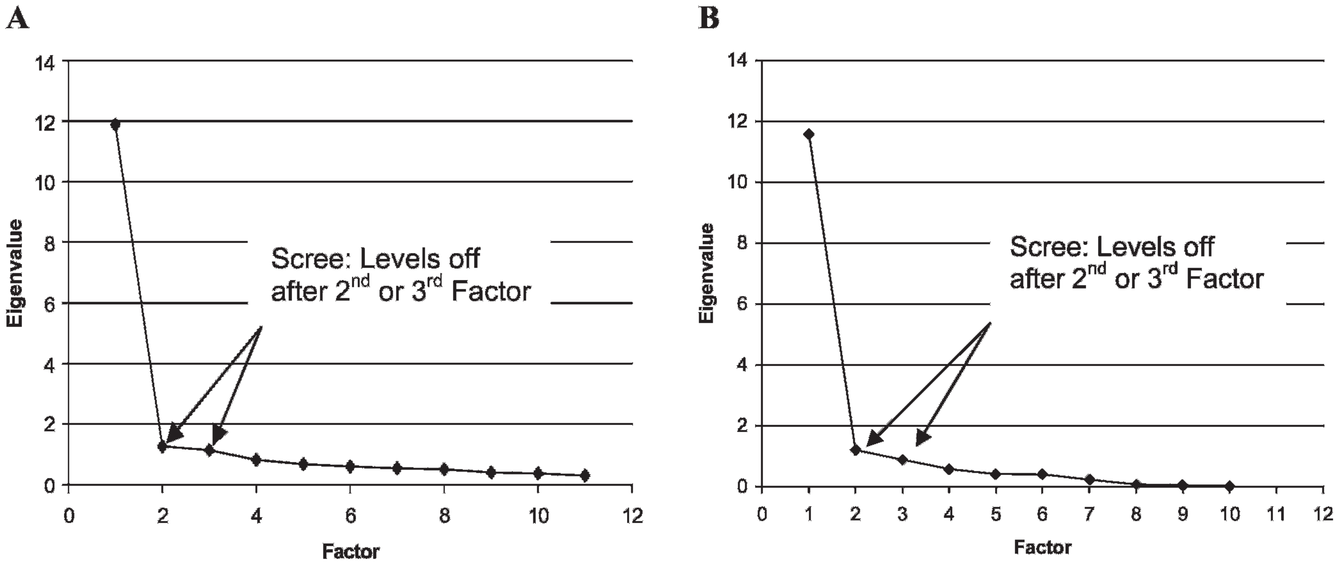

Figure 3. Cattell scree plots for (*A*) principal component analysis and (*B*) common factor analysis. Derived from a sample data set of 250 responses to a 20-item hypothetical questionnaire.

advantage to this approach is its statistical basis of retention rather than an arbitrary eigenvalue threshold (such as the Kaiser criterion). However, this method is sensitive to large sample sizes (800-1,000), which can lead to retention of too many factors.[25]

Authors should indicate a priori at least two approaches they will use to determine the number of factors to retain in a solution.[6] In addition, they should consider whether the retained factors make practical and theoretical sense.[1] When there is uncertainty about the number of factors to retain, authors are recommended to retain too many rather than too few.[22]

For the sample data set, the results of the Kaiser criterion, amount of variance explained and the Cattell Scree Test for PCA and CFA are provided in Table 2 and Figure 3. Note that the number of retained factors differs by factor extraction method. With PCA, the three factors with eigenvalues greater than 1.0 are retained using the Kaiser criterion. The decision to retain factors that account for at least 70 per cent of total variance accounts for the first three factors (71 per cent). The PCA Cattell Scree Test plot is more difficult to interpret, leveling off after the second or third factor (see Figure 3A). Together, these criteria suggest retaining three factors. With CFA, the two factors with eigenvalues greater than 1.0 are retained using the Kaiser criterion. The decision to retain factors that account for at least 60 per cent of total variance accounts for the first two factors (64 per cent). The CFA Cattell Scree Test plot is also difficult to interpret, leveling off after the second or third factor in the CFA solution (see Figure 3B). Together, these criteria suggest retaining two factors. Note how the suggested number of factors to retain may differ depending on the factor analytical technique. Hence, it is important for authors to determine their technique a priori to avoid any discrepancies related to the number of factors to retain.

Assessing the Factor Loading Matrix

The next step in the factor analytical process is to determine the extent to which items load on a given factor (see Figure 2). A factor loading matrix contains factor loadings, correlations of every item with each of the retained factors. High factor loadings indicate a strong relationship between an item and a given factor,[8] with values greater than 0.40 commonly defined as "loading" on that factor.[3,6,26] Items with no factor loadings above 0.40 contribute little to the overall questionnaire and, if clinically sensible, should be considered for deletion.[24] Factor loading matrices are easiest to interpret when each item loads heavily on only one factor. At least three items should load on each retained factor.[6] To achieve such criteria, further analyses of factor loading matrices are often required.

Using the sample data set, factor loading matrices for PCA and CFA demonstrate the correlations of each item in the questionnaire to each of the retained factors (Table 3). With both PCA and CFA solutions, all items load on factor 1, and two items (5 and 20) load on both factors 1 and 2. With the PCA solution, no item loads on factor 3. Because all items load on factor 1, these factor loading matrices are difficult to interpret.

Table 3. Factor Loading Matrices for Principal Component Analysis and Common Factor Analysis

	PCA: 3 Factors Retained		
Item	*Factor 1*	*Factor 2*	*Factor3*
1	**0.77**	−0.28	−0.15
2	**0.77**	−0.12	−0.21
3	**0.70**	−0.40	0.34
4	**0.76**	−0.37	0.31
5	**0.80**	**0.58**	−0.70
6	**0.72**	−0.08	0.16
7	**0.74**	−0.22	0.26
8	**0.78**	0.06	−0.33
9	**0.80**	0.14	−0.33
10	**0.77**	0.05	−0.12
11	**0.76**	−0.17	−0.22
12	**0.69**	0.37	−0.3
13	**0.82**	−0.13	−0.20
14	**0.82**	0.17	0.01
15	**0.83**	0.05	−0.17
16	**0.65**	0.32	0.32
17	**0.84**	0.13	0.14
18	**0.68**	0.11	0.32
19	**0.80**	−0.19	0.32
20	**0.79**	**0.48**	−0.50
Variance λ explained	**11.885***	**1.263**	**1.132**

	CFA: 2 Factors Retained	
Item	*Factor 1*	*Factor 2*
1	**0.77**	−0.27
2	**0.77**	−0.12
3	**0.69**	−0.33
4	**0.75**	−0.32
5	**0.79**	**0.62**
6	**0.71**	−0.05
7	**0.73**	−0.16
8	**0.78**	0.04
9	**0.80**	0.11
10	**0.76**	0.04
11	**0.75**	−0.17
12	**0.68**	0.29
13	**0.82**	−0.13
14	**0.81**	0.15
15	**0.83**	0.04
16	**0.65**	0.34
17	**0.83**	0.13
18	**0.67**	0.13
19	**0.79**	−0.16
20	**0.77**	**0.57**
Variance λ explained	**11.590***	**1.203**

CFA = common factor analysis; PCA = principal component analysis.
Factor loadings > 0.4 are bolded.
Factor loadings > 0.4 on more than one factor are bolded and shaded.
Derived from a sample data set of 250 responses to a 20-item hypothetical questionnaire.
*Eigenvalues: amount of variance in all items explained by Factor 1.

Factor Rotation

The goal of factor rotation is to assist with factor interpretation by achieving a structurally simple matrix compared with the original factor loading matrix.[27] In a simple factor loading matrix, each item loads on exactly one factor and each factor possesses high loadings for only some items (meaning that the amount of variance is distributed evenly among the factors).[10] Factor rotation is "the process of turning the reference axes of the factors about their origin to achieve a simpler structure and theoretically more meaningful factor solution" (p. 132).[3] Axes are rotated through different angles based on the variance of the factor loadings to maximize differences between high and low loadings of items on a given factor.

The two most common types of rotation are termed *orthogonal* and *oblique* (see Figure 2). Orthogonal rotation assumes that the retained factors are independent of one another (uncorrelated). The reference axes of the factors are rotated so that they remain fixed at right angles to each other.[28] Oblique rotation assumes that the factors are correlated. Each factor is rotated separately, allowing the axes to be drawn closer to a group of items. An oblique rotation will generally yield different communality estimates compared with the original factor loading matrix, whereas an orthogonal rotation will leave these values unchanged.

Despite advantages to orthogonal rotation, oblique rotation is often preferred.[3] Although orthogonal rotation facilitates factor interpretation because the explained variances no longer overlap (they are uncorrelated), the rotation may result in variance spread among less important factors, falsely implying the absence of a factor.[3] Oblique rotation is often considered a more accurate reflection of reality since correlation among factors is often expected.[8] Furthermore, the flexibility of oblique rotation can produce a structurally simpler matrix that is easier to interpret,[28] although the direction and degree of correlation between factors are often unknown. Whereas some suggest setting correlations among factors at 0.30,[1] there are no universal guidelines to indicate the optimal factor correlations for oblique rotation.

Although authors would normally choose one rotation a priori, both rotation results are illustrated with the sample data set to demonstrate the differences between orthogonal and oblique rotation (Table 4). In the PCA solution, 12 items load on two or more factors with orthogonal rotation (solution 1). However, with oblique rotation (solution 2), a simpler structure is achieved whereby only one item loads on more than one factor (item #20). In the CFA solution (see Table 4), 12 items load on two or more items with orthogonal rotation (solution 3). Similarly, with oblique rotation (solution 4), a simpler structure is achieved whereby only one item loads on more than one factor (item #20). The change in factor loadings between the types of rotation is attributed to the differences in the amount of variance explained by the factors.

Because the items load more uniformly across the factors, rotation results in a more evenly distributed variance compared with the unrotated solutions in Table 3. After rotation, at least three items load on each factor, satisfying the three-item factor loading minimum. The magnitude of factor loadings also appears either higher or lower after rotation. Oblique rotation results in a simpler factor loading matrix compared with orthogonal rotation because the factors are correlated, allowing the axes to be drawn closer to a group of items.

Factor Interpretation

Factor interpretation, the final step of factor analysis, is the assignment of a meaning and name to each factor according to the domain each represents (see Figure 2). Because higher factor loadings associated with an item represent a greater degree of overlap between that item and a given factor, items that load on a factor provide hints about the factor meaning.[26] Items that continue to load on more than one factor after rotation may suggest that they relate to more than one factor.

Even after rotation, authors may find items that load heavily on multiple factors, thus complicating interpretation. Although some suggest deleting items that load on multiple factors,[28] this may result in inadvertently removing important items. Authors should retain these items and allocate them to the factor that makes most clinical sense based on the construct being measured.[10,24] Others suggest using statistical tests of internal consistency to determine item placement.[3] Authors may also find items that do not load on any factors after rotation and decide to remove these items from the questionnaire. Other methods of item reduction may occur when two or more questionnaire items are conceptually very similar, resulting in the removal of unnecessary items to minimize redundancy. As with other steps, authors should state their preferred approach a priori and provide justification that their decisions for item reduction are clinically sound.

Once all items are allocated to a factor, the final stage of interpretation is to name the factors. Factor names should refer to the questionnaire domains that contribute to the

Table 4. Factor Rotation: Orthogonal and Oblique Rotated Factor Pattern Loading Matrices for Principal Component Analysis and Common Factor Analysis

	PCA					
	Orthogonal (Solution 1)			Oblique (Solution 2)		
Item	Factor 1	Factor 2	Factor 3	Factor 1	Factor 2	Factor 3
1	**0.64**	0.33	0.90	**0.63**	0.39	−0.19
2	**0.68**	0.39	0.19	**0.71**	0.19	−0.05
3	**0.82**	0.24	0.16	**0.91**	0.04	−0.04
4	**0.83**	0.30	0.19	**0.89**	0.03	−0.02
5	**0.60**	**0.58**	0.13	0.37	**0.55**	−0.13
6	**0.54**	**0.57**	0.35	0.19	**0.47**	0.20
7	**0.69**	**0.42**	0.28	0.08	**0.69**	0.11
8	**0.77**	**0.62**	0.30	0.09	**0.86**	0.08
9	**0.78**	**0.58**	0.36	0.15	**0.87**	0.16
10	**0.61**	**0.44**	0.36	0.12	**0.58**	0.18
11	**0.68**	**0.41**	0.14	0.22	**0.72**	−0.11
12	**0.47**	**0.61**	0.31	0.23	**0.41**	0.12
13	**0.71**	**0.43**	0.22	0.22	**0.72**	−0.04
14	**0.54**	0.36	**0.53**	0.34	0.14	**0.43**
15	**0.69**	0.34	**0.47**	0.17	0.09	**0.68**
16	0.18	0.18	**0.89**	−0.08	0.00	**0.97**
17	0.35	0.27	**0.56**	0.27	0.31	**0.44**
18	0.20	0.26	**0.83**	−0.07	0.10	**0.88**
19	0.31	0.36	**0.74**	0.02	0.20	**0.75**
20	**0.56**	**0.47**	**0.44**	**0.49**	**0.44**	**0.42**

	CFA			
	Orthogonal (Solution 3)		Oblique (Solution 4)	
Item	Factor 1	Factor 2	Factor 1	Factor 2
1	**0.76**	0.28	**0.82**	−0.01
2	**0.67**	0.40	**0.65**	0.18
3	**0.75**	0.19	**0.84**	−0.12
4	**0.79**	0.23	**0.87**	−0.09
5	**0.78**	**0.41**	**0.83**	0.01
6	**0.58**	**0.52**	**0.53**	0.23
7	**0.67**	**0.53**	**0.67**	0.10
8	**0.57**	**0.63**	**0.47**	0.39
9	**0.54**	**0.59**	**0.48**	0.38
10	**0.56**	**0.51**	**0.47**	0.37
11	**0.69**	**0.45**	**0.69**	0.11
12	**0.44**	**0.65**	**0.65**	0.12
13	**0.71**	**0.42**	**0.69**	0.19
14	**0.53**	**0.63**	0.37	**0.53**
15	**0.62**	**0.55**	0.40	**0.51**
16	0.14	**0.85**	−0.22	**0.99**
17	0.36	**0.63**	0.41	**0.52**
18	0.22	**0.79**	−0.09	**0.88**
19	0.31	**0.71**	0.14	**0.71**
20	**0.54**	**0.56**	**0.42**	**0.44**

CFA = common factor analysis; PCA = principal component analysis.
Factor loadings > 0.4 are bolded.
Factor loadings > 0.4 on two or more factors are bolded and shaded.
Derived from a sample data set of 250 responses to a 20-item hypothetical questionnaire.

overall measurement of the construct and should capture the themes represented within the items that are grouped together. Items that load heavily on factors,[3] as well as background theory, may provide clues to potential factor names.[10]

The sample questionnaire may consist of three or two domains, depending on the factor analytical approach (PCA or CFA). Items that "belong" with each domain are indicated in Table 4. In the PCA oblique solution (solution 2), items 1 to 4 load on factor 1, items 5 to 13 load on factor 2 and items 14 to 19 load on factor 3. Item 20 loads on all three factors. Factor 1 includes items such as pain, energy level and strength. Factor 2 includes items such as walking, eating, laundry, cleaning and groceries, ability to work and taking transportation. Factor 3 includes items such as feeling sad, afraid, forgetfulness or concentration. Based on common themes in the respective items, factors 1, 2 and 3 may be named "physical impairments," "activity limitations and/or participation restrictions" and "mental impairments," respectively. In the CFA oblique solution, (solution 4), items 1 to 13 load on factor 1 and items 14 to 19 load on factor 2, indicating two potential factors that may be termed "physical health" and "mental health," respectively. Item 20 asks about "general health" and loads on all three factors in both solutions. This is because general health may refer to both the physical and mental components of health. In this situation, it may be acceptable for an item to load on multiple factors given their dual meaning.

Sample Size, Missing Data and Cross-Validation

Other features that should be considered when reviewing an article on factor analysis include sample size, missing data and cross-validation. Sample sizes should include 5 to 10 patient responses for every item in a questionnaire,[5,22] with a minimum of 100 responses.[28] Missing data should not exceed more than 25 per cent per item. When missing data are assumed to be random, imputation of mean values to the data set is acceptable.[29] Finally, if a data set is large enough, authors may choose to validate the factor analytical findings using cross-validation, in which the factor analytical results of one part of a data set are compared with another part of the same data set. For example, authors may perform an exploratory factor analysis on half of the data and confirmatory factor analysis on the other half.[29]

The sample data set consists of 250 complete, subjective patient responses to 20 items in the questionnaire. Because the sample size is 12 times the number of items in the questionnaire, the data set satisfies the minimum 1:5 item to patient response ratio.[5,22] Furthermore, missing data for the items ranged from 16 to 20 per cent for 19 of the 20 items, which is below the 25 per cent item threshold, making it safe to conduct the factor analysis. Cross-validation was not performed for this example.

CONCLUSIONS

Factor analysis is an important statistical technique in the field of measurement that identifies interrelationships among a set of items and groups them into homogeneous domains. Factor analysis can be used to reduce the number of items in a questionnaire, determine whether a data set can be explained by a smaller number of domains or test a hypothesis about the underlying structure of a data set. Identifying the domain structure of a questionnaire using factor analysis may help clinicians identify relevant questionnaires to administer with clients and determine individual client component or domain scores. This may assist clinicians in highlighting certain areas of impairments, activity limitations or participation restrictions experienced by an individual.

Using a sample data set, this article highlights the complexity of the factor analytical process to consider when reviewing an article using factor analytical techniques. Although there is no standardized method to direct this process, authors should clearly justify and outline a priori their purpose for using factor analysis, the conceptual approach, preferred technique and methods that guided their decision-making throughout. Additionally, authors should consider validating their factor analytical results using confirmatory approaches to enhance confidence in their findings. Finally, after using factor analytical techniques, authors should indicate any future work needed to assess measurement properties of newly developed questionnaires.

GLOSSARY

Amount of variance explained: A method used to determine the number of factors to retain, suggesting that factors should be retained until a predetermined threshold of variance is attained (e.g., extract those factors that account for 90 per cent of the explained variance), or until the last factor accounts for a small proportion of the explained variance (less than 5 per cent).

Cattell Scree Test: A method that uses a plot of eigenvalues against each factor to determine the number of factors to retain. The test suggests that factors prior to the scree part of the plot (the part where the slope changes from negative to almost zero) are retained in the factor analytical solution.

Communality: A feature of items in a data set that represents the amount of variance each individual item has in common with all other items. Communalities range between 0 and 1.0 High final communalities that are close to 1.0, calculated from the results of a factor analysis, suggest that factors explain the variance better than the individual items themselves.

Component: A linear combination of items that represents an underlying domain. The term *factor* is used in reference to CFA and the term *component* in reference to PCA.

Construct: An abstract concept of interest being measured (e.g., health).

Domain: An item or group of items in a questionnaire that relate to one another.

Data set: A collection of observed variables or responses to items from a number of individuals.

Eigenvalue: A feature of factors defined as the amount of variance in all items explained by a given factor or component. Factors with larger eigenvalues account for greater variance compared with factors with lower eigenvalues. PCA eigenvalues are estimates of the total amount of variance. CFA eigenvalues are estimates of the common variance among items accounted for by a given factor.

Factor: A linear combination of items that represents an underlying domain. The term *factor* is used in reference to CFA and the term *component* in reference to PCA. For this article, the term *factor* is used to refer to both a CFA factor and the analogous PCA component.

Factorable: Refers to a correlation matrix that is statistically "safe" to proceed with the factor analytical process.

Factor analysis: A statistical technique that groups individual questionnaire items or variables into homogeneous domains.

Factor loadings: Correlations of every item with each factor. Loading values > 0.40 indicate that an item "loads" or is related to that factor.

Factor loading matrix: A matrix consisting of factor loadings.

Factor rotation: A step in the factor analytical process used to transform the factor loading matrix into a structurally simpler matrix to make it more easily interpretable. Orthogonal rotation assumes that factors are uncorrelated with each other (e.g., physical health factor is uncorrelated with mental health factor), whereas oblique rotation assumes that factors are correlated.

Initial communalities: Preliminary estimates used during the analysis that estimate the amount of variance each individual item has in common with all other items. Initial PCA communalities represent the total variance of each item (common and unique) and are assumed to equal 1.0. Initial CFA communalities represent a proportion of the total variance (common) and are estimated as being less than 1.0.

Item: Question in a questionnaire or item in a data set.

Kaiser criterion: A method used to decide the number of factors to retain that suggests that factors with eigenvalues greater than 1.0 (which account for more of the total variance than a single item) should be retained in the factor analytical solution.

Proportion of variance explained: Proportion of variance explained by a given eigenvalue, calculated by dividing a given eigenvalue by the total variance.

Questionnaire: A question or set of question(s) or item(s) that measure a construct of interest. Questionnaire may also refer to a scale, instrument or self-report tool.

Structure: The interrelationships among variables in a data set. Structure may also refer to the overall framework of a questionnaire.

Structurally simple matrix: A factor loading matrix whereby each item loads on exactly one factor and each factor possesses high factor loading values for only some items.

Variable: Quantity that can assume any set of values.

Variance: Variability among items in a data set. Common variance is shared among a set of items that can be explained by a common set of factors. Unique variance includes variance specific to an item and not explained by common factors plus error variance, which is attributed to random measurement error

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